Appendix A, Crystal Lake
Estimating Phosphorous Loading Using Reckhow's Eutromod Loading Equations for Lakes

Equations utilizing regression analysis have been developed by Kenneth Reckhow for the EUTROMOD Watershed and Lake Model. These equations have been incorporated into the Crystal Lake Restoration Diagnostic/Feasibilty Study to compare total phosphorous concentration values computed using the model equations and those obtained by direct measurement. Information required to use the model equations includes average total phosphorous influent concentration, 0.2 mg/l; mean depth, 1.5 m; and the hydraulic residence time, 0.40 yr.

Calculation of the predicted total phosphorous load is as follows:

$$k = 10.77\tau^{0.39} z^{0.01} P_{in}{}^{0.82} = 10.77(0.40^{0.39})(1.5^{0.01})(0.2^{0.82}) = 2.021$$

$$\log_{10}(P) = \log_{10}\left[\frac{P_{in}}{1+k\tau}\right] = \log_{10}\left[\frac{0.2}{1+(2.021)(0.40)}\right]$$

$$P = 0.11 mg/l$$

These two equations are from the *EUTROMOD Watershed and Lake Modeling Software* documentation and are described below.

EUTROMOD – KA, MO, OK, AK, IA, NE Lake Models
K. H. Reckhow
Duke University, June 10, 1991

MODEL PARAMETER ESTIMATION

The models have been fitted using robust regression (linear, nonlinear, and logistic). The robust weighting scheme usually results in a model that best (in a least squared error sense) represents the pattern in the bulk of the data. Standard errors have been adjusted to account for the robustness criterion.

MODELS

Total Phosphorous (mg/l)

$$\log_{10}(P) = \log_{10}\left[\frac{P_{in}}{1+k\tau}\right]$$

Where $k = 10.77\tau^{0.39} z^{0.01} P_{in}^{0.82}$ standard error = 0.219

Total Nitrogen (mg/l)

$$\log_{10}(N) = \log_{10}\left[\frac{N_{in}}{1+k\tau}\right]$$

Where $k = 10.77\tau^{0.39} z^{0.01} P_{in}^{0.82}$ standard error = 0.108

Chlorophyll a (μg/l)

$$\log_{10}(Chla) = 1.99 + 0.51\log_{10}(\hat{P}) + 0.23\log_{10}(\tau) - 0.35\log_{10}(z)$$

standard error = 0.226

Secchi Disk Depth (m)

$$\log_{10}(SD) = -1.32 - 0.66\log_{10}(\hat{P}) + 0.47\log_{10}(z)$$

standard error = 0.171

Trophic State Index

The TSI is based on that proposed by R, Carlson (1977) using predicted P, Chla, and SD. Allowable Phosphorous loading to meet in-lake goal:

$$\log_{10}(P_{in}) = \log_{10}\left[P\left(1 + 1{,}49\tau^{0.35} z^{-0.97} p^{-0.78}\right)\right]$$

standard error = 0.317

SYMBOLS

P_{in}, N_{in} = average influent concentrations (mg/l)
τ = hydraulic detention time (year)
$\hat{P}$ = predicted in-lake phosphorous concentration (mg/l)
$\hat{N}$ = predicted in-lake nitrogen concentration (mg/l)
Z = lake mean depth (m)
Log_{10} = base 10 logarithm

CONSTRAINTS ON ABOVE MODELS

The following constraints reflect the data set used to fit the models. In some instances (e.g., nutrient retention less than zero) additional constraints were imposed to create homogeneity in the data set or to eliminate suspected errors. Constraints involving phosphorous refer only to models that include phosphorous; constraints involving nitrogen refer only to models involving nitrogen.

Phosphorous Retention (R_P) > zero
0.003 mg/l < P < 0.424 mg/l
0.010 mg/l < P_{in} < 1.334 mg/l
Nitrogen Retention (R_N) > zero
0.090 mg/l < N < 7.185 mg/l
0.268 < N_{in} < 10 mg/l
0.0008 mg/l < Chla < 0.953 mg/l
0.008 year < τ < 285 year
1.2 m < z < 3.6 m